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ON THE DECREASE OF THE RADAR CROSS SECTION OF THE APOLLO COMMAND MODULE DUE TO REENTRY PLASMA EFFECTS

JOHN W. MARINI

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John W. Marini

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ABSTRACT

During reentry, the radar cross section of a space vehicle may decrease due to a number of effects associated with the plasma sheath. The decreases occur at certain combinations of altitude and speed. In this report, some regions of decrease are determined for the case of the Apollo Command Module approaching a C-band radar.

SUMMARY

A knowledge of the radar cross section of the Apollo Command Module is important in connection with reentry acquisition and skin-tracking. It is known from actual reentry tests, however, that effects associated with the reentry plasma sheath can cause large decreases from the free space cross section of reentering objects.

A number of mechanisms have been proposed to explain the decreases that have been observed. Among these are, first, a diverging lens type of effect where a photo-ionized cloud preceding the sheath acts to divert the incident radar signal around the object. Second, the plasma sheath, at certain combinations of altitude and speed, may act as an absorbant coating. Third, the inhomogeneous plasma sheath may present to the incident radar wave a reflecting surface having, because of its shape, a lower cross section than the original body surface.

With the reentry object, radar frequency, and radar look-angle fixed, the regions where these mechanisms are likely to have effect depends primarily on the altitude and speed of the object. In this report, the regions of decrease due to the last two of the mechanisms listed above are determined. The case considered is that of a C-band radar observing a large directly-approaching object.

Neither the possibility of a decrease due to photo-ionization nor the effect of the sheath on a radar observing the object at other than a head-on view have yet been treated. Because of the incomplete and tentative nature of the analyses and because of the lack of field data on reentries of the Apollo type, it is premature to say whether the effects constitute a practical tracking problem.

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INTRODUCTION

The subject of this report is the decrease in radar cross section that occurs during the atmospheric reentry of space vehicles. Some of the theories of the cause of cross section decrease are described, and some approximate bounds on the occurrence of decreases are given. The bounds are intended for use in connection with the reentry of the Apollo Command Module, and the considerations are limited, for the most part, to the cross sections of large blunt objects observed head-on or nearly so.

In returning from a lunar mission, the Apollo Command Module undergoes a sustained flight in the upper atmosphere, landing several thousands of miles from the reentry point (Reference 1). At the same time the Module must conduct appropriate maneuvers to confine its trajectory to a safe flight corridor. Since the trajectories lying within the flight corridor may be separated by hundreds of miles, the tracking of the Command Module during the reentry period is especially critical.

Radio transmission between the ground and a space vehicle is, in general, degraded or interrupted by the plasma sheath that forms about the vehicle as it reenters the atmosphere. In the case of the Apollo Command Module returning from a lunar mission, both S-band (2GHz) and also frequencies as high as C-band (5GHz) will be totally blacked-out over a large part of the reentry trajectory (Reference 2). Consequently, during this period, active tracking of the vehicle through the use of an onboard transponder is not feasible using C-band or lower frequencies.

The NASA reentry ships, the Huntsville and the Watertown, are equipped with radars operating at C-band. These will be used for passive acquisition and skin-tracking of the reentering Apollo Command Module (Reference 3).

The success of acquisition and skin tracking depends, however, on the existence of a target of adequate radar cross section. While the free-space cross section of the Module is adequate for this

purpose (Reference 3), a substantial decrease in cross section, such as might be caused by reentry plasma effects, could result in failure to acquire or in loss of track. Consequently, it is important to be able to predict both the region along a reentry trajectory where decreases in cross section might occur and also the magnitude of these changes. With this knowledge, it might be possible to locate the reentry ships in such a way as to avoid configurations where decreases in radar cross section deep enough to interfere with radar performance take place during critical portions of the tracking period.

A complete solution to the problem of quantitative prediction of cross section changes along a given trajectory is beyond the state-of-the-art at this time. In this report some of the regions where decreases in cross section should occur are estimated. No attempt is made to determine the magnitude of the decrease.

BEHAVIOR OF THE RADAR CROSS SECTION OF A REENTERING OBJECT

The general features of the behavior of the radar cross section of a reentering object have been known for a number of years both from observations on reentering missiles (References 4 and 5) and from experimental programs such as the Trailblazer (Reference 6).

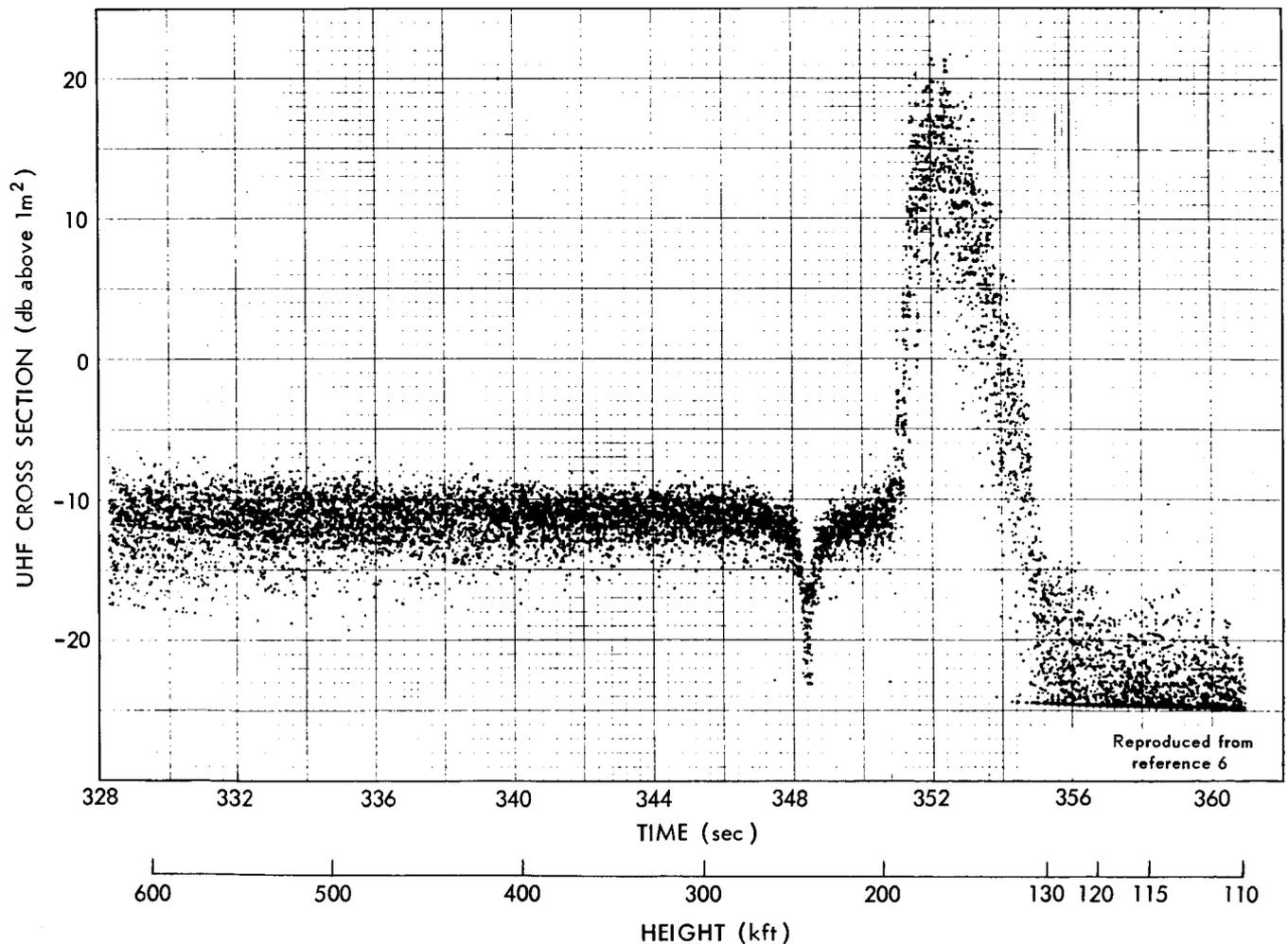


Figure 1—UHF Cross Section as Function of Altitude for Trailblazer 1k.

Above the earth's atmosphere, of course, the free space cross section of the reentering object is seen. As the object descends, a high altitude decrease in cross section is often observed at heights of roughly 250,000 feet. Following this dip, a period of relatively constant radar return may occur due, presumably, to the formation of a strongly-reflecting ionized sheath about the object. Next, the presence of a wake of sufficiently strong ionization results in a marked enhancement of the reflected radar signal. As reflections from the wake disappear at, say, 150,000 feet, a second decrease of radar cross section below the free space value is sometimes observed. Finally, the plasma sheath effects disappear altogether and free-space cross section of the object is seen again.

It should be pointed out that the description above is often violated. The decreases in cross section may or may not be seen and the changes that do occur sometimes exhibit anomalous behavior (Reference 7).

Some of the above effects were observed in the Trailblazer program. In Figures 1 and 2 (reproduced from Reference 6), the radar cross section of Trailblazer 1k is shown as a function

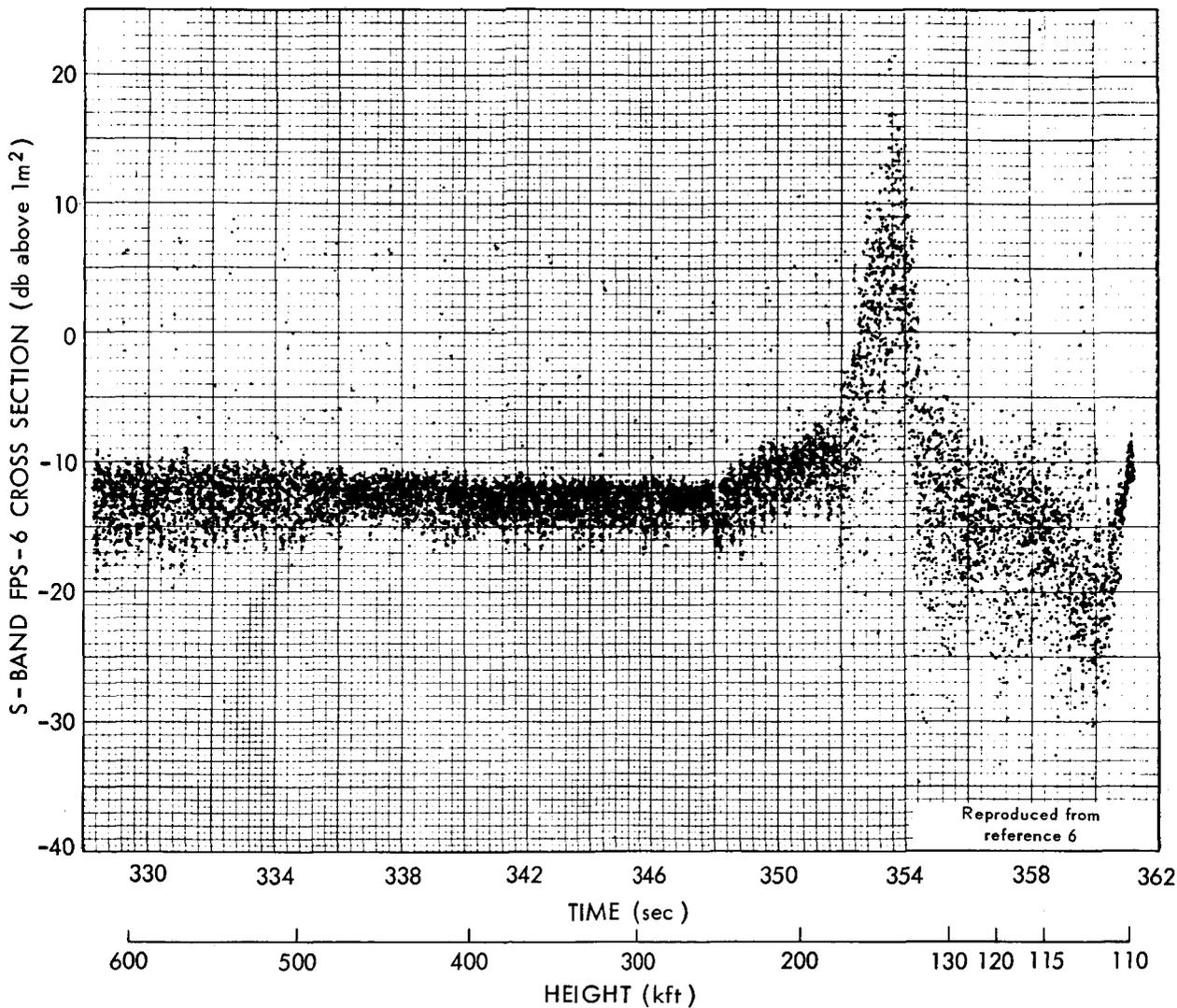


Figure 2—S-Band Cross Section as a Function of Altitude for Trailblazer 1k.

of altitude. Trailblazer 1k was an eight inch diameter aluminum sphere with a reentry speed of about 19,000 feet per second (Reference 6). The UHF radar operated at 420 MHz, and viewed the

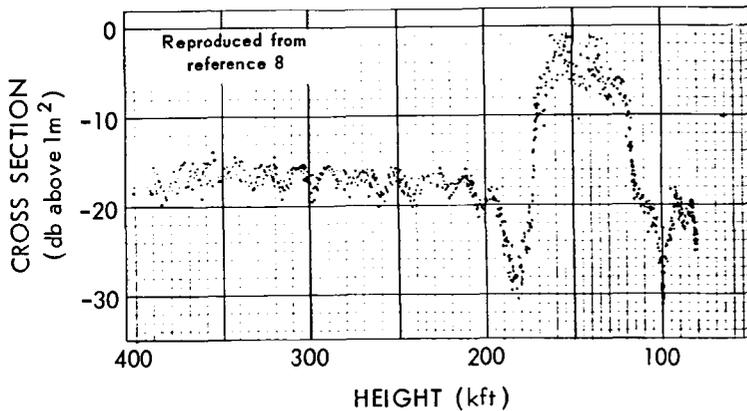


Figure 3—Trailblazer 1b cross section at UHF vs height.

sphere at an angle of about 45 degrees with the velocity vector. As the sphere descended, the free space cross section was measured until an altitude of 240,000 feet was reached at which point an order of magnitude decrease in cross section took place. This was followed, after a period, by a large enhancement due to the wake. Figure 2 shows a simultaneous observation with an S-band (2800 MHz) radar. The pronounced high-altitude decrease at 240,000 feet has vanished, although there is possibly some indication of a small dip at the same altitude. In Figure 3 (reproduced from Reference 8), the UHF cross

section of the slower (14,000 feet per second) Trailblazer 1b is shown. Here the high altitude decrease is immediately adjacent to the wake enhancement, and a low altitude decrease also adjacent to the wake, is evident.

THEORIES OF RADAR CROSS SECTION DECREASE

The exact calculation of the backscatter cross section of a reentering object is a very difficult problem. In principle, the procedure for making this calculation would be, first, to determine the configuration and properties of the plasma sheath and surrounding atmosphere. With the electron density and collision frequency in the plasma determined thereby, the small-signal electromagnetic properties of the sheath are specified and the distribution of the complex dielectric constant about the object is determined. Given this distribution and the shape and material of the object, Maxwell's equations with appropriate boundary conditions would have to be solved.

As a practical matter, the determination of the properties of the sheath is, in itself, an enormously complicated problem. Moreover, even if the complete solution to this problem were in hand, the exact solution to Maxwell's equation would not be feasible because of the complex geometry involved.

In situations of this nature, the usual practice is to resort to the study of relatively simple models which, it is hoped, will exhibit the main features of the original problem. Three such models, each of which appears to have its own range of validity, are described below.

In the first model, the three dimensional character of the reentering object is retained, but the variation of plasma and object properties with angle are ignored. The model of the reentering object is a conducting sphere surrounded by a plasma whose properties likewise are independent of both

angular coordinates. With the variation in electromagnetic properties of the model occurring only in the radial direction, an analytical treatment of the problem becomes feasible. Peters & Green (Reference 9) have studied the echo area of a perfectly conducting sphere coated with a homogeneous collisionless plasma layer using optical approximations. An important feature brought out by this study is that when the radar frequency illuminating this target exceeds the plasma frequency of the layer, the layer acts as a diverging lens to reduce the apparent cross section as shown in Figure 4. This occurs despite the fact that the plasma is assumed collisionless, and consequently, absorbs no energy.

Murphy, also, has worked with the spherical model (References 8 and 10). He emphasizes the important possibility that a significant cause of cross section decrease may be not necessarily the shock-induced plasma sheath about the reentering object but rather an ionized cloud in front of the shock wave produced by photo-ionization. In addition, he considers the case of wavelengths larger than the dimensions of the body and plasma coating. In this regime, optical concepts, i.e., the diverging lens concept, are no longer valid. Murphy shows, nevertheless, that backscatter reduction may be caused by a collisionless plasma at these wavelengths due to the phenomena of plasma resonance (Reference 11).

A second model for decrease of radar cross section disregards the shape of the object altogether, and focusses its attention on the properties of the plasma. In this case, the object can be modelled as an infinite perfectly-conducting plane coated with a plasma slab. The model is illustrated in Figure 5. Here, the collision frequency of the plasma plays a vital part since the power incident on the conductor-backed plasma layer must eventually be totally reflected unless some mechanism for absorbing energy exists in the plasma.

The properties of absorbent layers such as shown in Figure 5 have been studied in connection with the use of coatings of absorbent materials to reduce radar cross section for purposes of

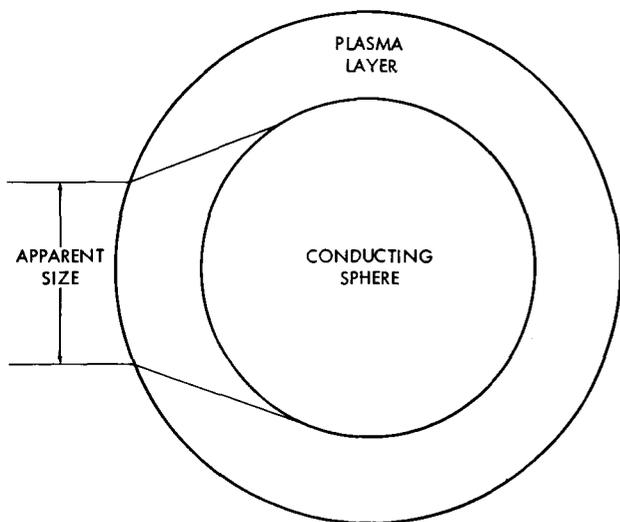


Figure 4—Decrease in the cross section of a sphere caused by a plasma layer acting as a diverging lens.

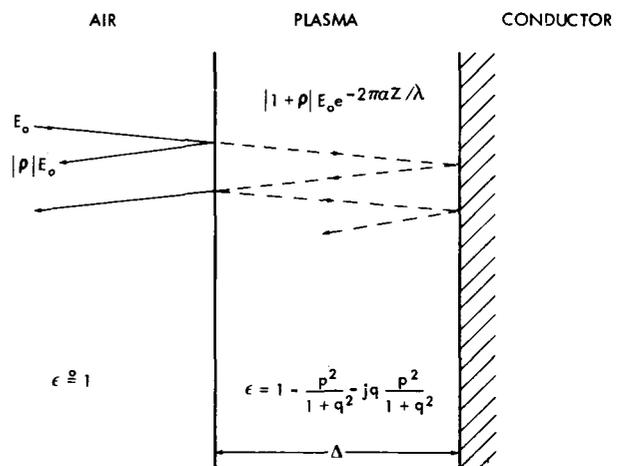


Figure 5—Absorption by a plasma layer. The magnitudes of the electric vectors of the incident wave and the first components of the reflected and transmitted waves are shown.

camouflage (Reference 12). Absorbers are considered to be of two kinds. In the interference absorber, the reflected wave at the layer-air interface is cancelled by destructive interference with the waves transmitted through the interface and reflected by the object. In the absorber of the second kind, the material is designed to eliminate reflection at the interface, while the wave transmitted into the layer and subsequently reflected by the body is eliminated by attenuation as it propagates through the layer. While an homogeneous plasma cannot act as a perfect absorber of the second kind because its relative permeability is unity (see Reference 12, Equation 14), absorption of the second kind can be obtained in a plasma for practical purposes, provided that the reflection coefficient is small, and that the plasma slab is sufficiently thick to produce substantial attenuation of the transmitted wave.

Musal (Reference 13) has studied in considerable detail the behavior of a plasma layer acting, principally, as an interference absorber, both for the simple case of a plane plasma slab and for more elaborate models.

The third model, one investigated by Blore and Musal (References 14, 15, and 16), concentrates on the property ignored in the first two models, the variation with angle of the properties of the sheath about the object. Consider an axisymmetric blunt body in space viewed head-on by a radar along the axis of symmetry. The scattered return from the body may be estimated by alternately adding and subtracting the returns from Fresnel half-period zones (Reference 17) drawn on the body surface. If the body enters the atmosphere at zero angle of attack the plasma sheath that forms has its maximum electron density in the vicinity of the stagnation point which lies in the center of the first and most important Fresnel zone. The electron density falls off rapidly as one proceeds away from this region around the body. Thus, the plasma sheath may, depending on the radar frequency and the values of electron density, reflect the incident wave at the front of the body, but be transparent to the wave farther back. In this way, the plasma sheath effectively deforms the shape of the body, and the basic model can be taken to be a conducting sphere covered by a metal cap*. Since the deformation initially takes place in and spreads out from a region (the first Fresnel zone) especially critical in its contribution to the calculated cross section, even a relatively thin plasma sheath (or cap) can cause a substantial cross section reduction.

On the basis of this model, the radar cross section has the nature of an interference effect taking place between the wave reflected from the cap and that reflected from the rest of the body. It consequently should be quite sensitive to changes in frequency or radar look-angle**, with the largest decrease taking place at small values of the angle, i.e., when the spacecraft is approaching the radar directly.

If a blunt body enters the atmosphere at an angle of attack, the symmetry in the example considered above is absent. However, if the body is large with respect to the radar wavelength, and is viewed approximately along the velocity vector, the distribution of the electron density in the plasma

*The capped metal sphere, is an adequate model for bringing out the basic physical concept involved. Blore and Musal (Ref. 16), however, actually use a physical optics method that takes into account the plane-wave reflection coefficient of the plasma sheath along the entire illuminated surface to calculate the radar cross section.

**Defined here as the angle made at the spacecraft between the spacecraft velocity vector and the line segment joining the spacecraft to the radar.

layer near the stagnation point may possibly exhibit sufficient symmetry about the radar-object line of sight to produce a cross section reduction due to this mechanism (see Figure 6, taken from Ref. 18).

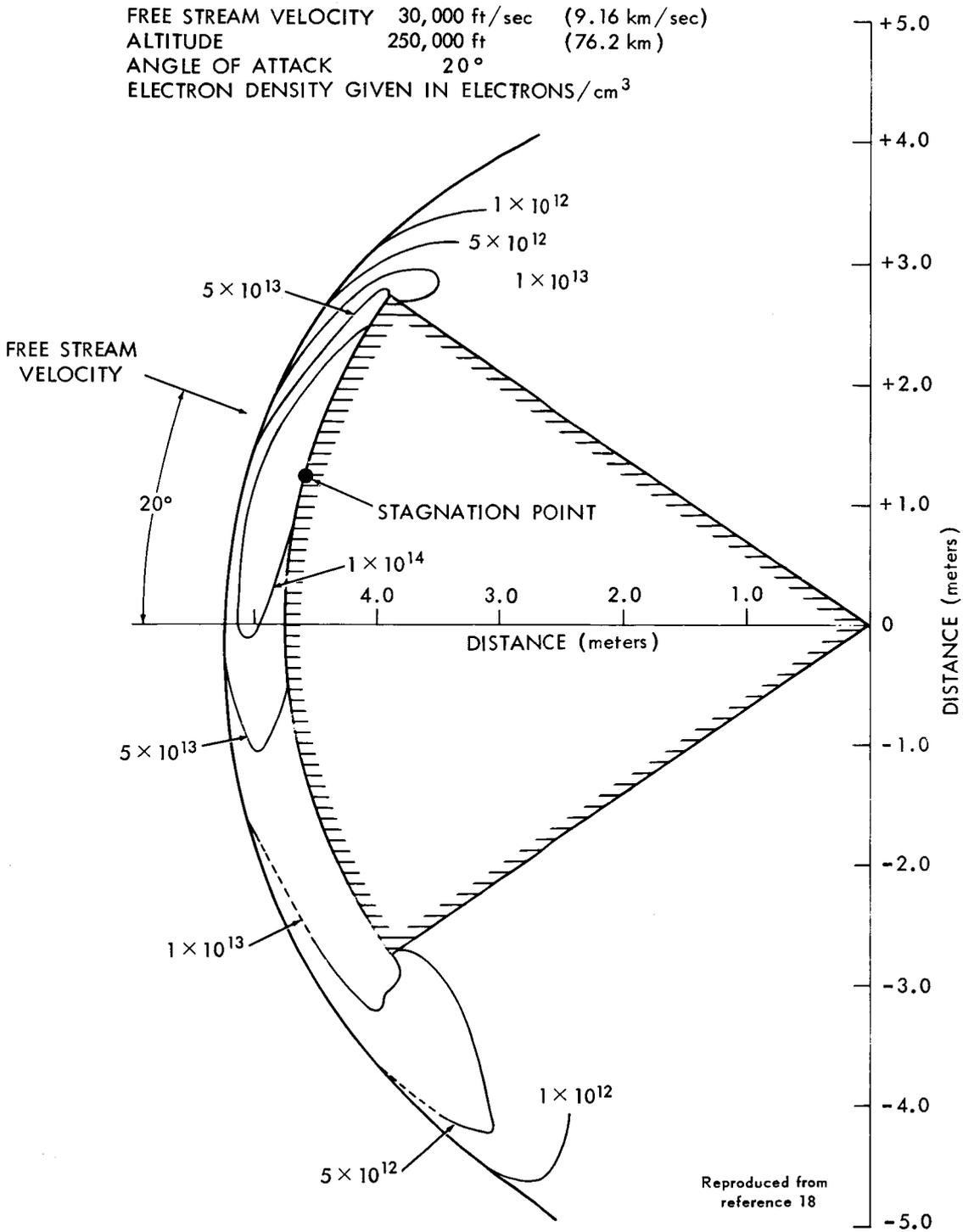


Figure 6—Electron Density Distribution in Plane of Symmetry.

While no single one of these simple models is sufficient, by itself, to explain all of the observed data, many of the observations may be explained by selecting the model that seems appropriate to the circumstances. For example, Murphy's explanation of the Trailblazer cross section reduction as an effect due to photo-ionization seems to fit the facts in this particular case. The electron density due to photo-ionization could well have been large enough to interact with the waves at UHF frequencies, but not dense enough to have caused an effect at S-band. Moreover, neither of the other models can explain the dip without also assuming photo-ionization, for the plasma sheath is too thin with respect to the wavelength for appreciable absorption to take place, and the body itself is too small for Fresnel zone type interference effects to occur. On the other hand, the radar cross section reduction measured by Blore and Musal (Reference 16) in a ballistic range could not be explained by photo-ionization, as the 35 GHz radar frequency was too high to be effected by the photo-ionization present. Also the shock stand-off distance was too small to permit appreciable absorption. Here, however, the calculated reduction using the "deformation" model agrees quite well with the measurements. Finally, the radar cross section decrease predicted using absorption theory has been shown, on occasion, to give agreement with actual reentry measurements (Reference 19).

Tentatively, each of these models will be accepted as a valid potential source of radar cross section decrease during reentry.

PLASMA RELATIONS

Bounds on the occurrence of radar cross section decrease caused by two of the models described above are derived in the following sections. The necessary background and formulas for the electromagnetic properties of a plasma are given in this section.

The simplest way to describe the electromagnetic properties of a plasma is in terms of its relative permeability and relative complex dielectric constant (Reference 20). The relative permeability of a plasma can be taken equal to unity. Using a time variation $\exp +j2\pi ft$ the relative complex dielectric constant of a plasma in the absence of a magnetic field* is given by (Reference 21)

$$\epsilon = 1 - \frac{p^2}{1 + q^2} - j q \frac{p^2}{1 + q^2} \quad (1)$$

where $p = f_p/f$ is the plasma frequency f_p normalized with respect to the radar frequency f and where $q = \nu/2\pi f$ is the normalized collision frequency.

The plasma frequency (in Hertz) can be determined from the electron density in the plasma through the equation

$$f_p = 9.0 \times 10^3 \sqrt{N_e} \quad (2)$$

*Under the conditions considered here, the presence of the earth's magnetic field can be neglected.

The collision frequency ν can be estimated using*

$$\nu = 1.8 \times 10^{-8} (T/300)^{1/2} N_a + 6.1 \times 10^{-3} (T/300)^{-3/2} N_i \quad (3)$$

where the electron density N_e , the neutral particle density N_a , and the ion density N_i are all expressed in particles per cubic centimeter, and the temperature T is in degrees Kelvin. The normalized frequencies p and q are calculated by dividing (2) and (3) by f and $2\pi f$ respectively, with f expressed in Hertz.

Two quantities useful in estimating the effect of the plasma sheath on radar cross section are the propagation constant of a uniform plane wave in a plasma and the basic plane wave reflection coefficient at the air-plasma interface.

The propagation of a uniform plane wave through a plasma is characterized by the formula

$$E = E_0 e^{-jn \ 2\pi z/\lambda} = E_0 e^{-\alpha \ 2\pi z/\lambda} e^{-j\beta \ 2\pi z/\lambda} \quad (4)$$

In the formula above, E represents one of the components of the wave, z is the distance travelled, λ is the free space wave-length, and n is the complex index of refraction

$$n = \beta - j\alpha = \sqrt{\epsilon} \quad (5)$$

If (1) is substituted into (5), α and β are found to be given by

$$\alpha = \left[\frac{1}{2} \sqrt{(1-S)^2 + q^2 S^2} - \frac{1}{2} (1-S) \right]^{1/2} \quad (6)$$

$$\beta = \left[\frac{1}{2} \sqrt{(1-S)^2 + q^2 S^2} + \frac{1}{2} (1-S) \right]^{1/2} \quad (7)$$

where the abbreviation

$$S = \frac{p^2}{1 + q^2} \quad (8)$$

*More refined equations exist. See, for example, reference 19, para. 3.3.

has been used. The attenuation in db per centimeter of a uniform plane wave propagating in a plasma is then given by

$$A(\text{db/cm}) = 40\pi \log_{10} e \cdot \alpha f / (3 \times 10^{10}) = 1.8 \times 10^{-9} \alpha f \quad (9)$$

The basic formula for the reflection coefficient of a plane wave propagating in one medium and normally incident on the boundary of a second medium is (Reference 20, para. 7.09)

$$p = (\eta_2 - \eta_1) / (\eta_2 + \eta_1) \quad (10)$$

where η_1 and η_2 are the intrinsic impedances of the first and second medium respectively. If the first medium is free space (or air) while the second medium is a plasma, the formula reduces to

$$\rho = (1 - \sqrt{\epsilon}) / (1 + \sqrt{\epsilon}) \quad (11)$$

Substituting (5) into (11) and multiplying the resulting expression by its complex conjugate, the power reflection coefficient R can be obtained

$$R = |\rho^2| = [(1 - \beta)^2 + \alpha^2] / [(1 + \beta)^2 + \alpha^2] \quad (12)$$

In decibels

$$R(\text{db}) = 10 \log \left[\frac{(1 - \beta)^2 + \alpha^2}{(1 + \beta)^2 + \alpha^2} \right] \quad (13)$$

Equations (13) and (9) are fundamental to the considerations of radar cross section of a plasma coated body. Equation (13) gives the fraction of incident power that is reflected initially at the air-plasma interface (assuming a plane-wave model), while equation (9) describes how rapidly the remaining fraction of the power is attenuated as it propagates through the plasma layer toward the surface of the object and then back toward the air after reflection from the object.

ALTITUDE-SPEED PLOTS

In order to apply the formulas listed in the previous section, the properties of the plasma, specifically, the distribution of the plasma frequency and the collision frequency about the body, must be known. Since the scope of this report is limited to small radar look-angles, the region of

primary interest is the portion of the plasma layer covering the nose of the object. The detailed configuration of the plasma sheath about the Apollo Command Module at various altitudes and speeds is presently being calculated by the Cornell Aeronautical Laboratory under NASA Contract NAS5-9978. Pending the completion of this work, the properties of the plasma layer in the region of interest will be taken to be those behind an equilibrium normal-shock flow. Although this assumption is not very good under most reentry conditions (Reference 22), it at least provides a quantitative basis for proceeding with the calculations that follow.

In Figure 7 the plasma frequency and collision frequency of equilibrium normal-shock flow as a function of altitude and speed are plotted. The data for these plots was taken from a report by Huber (Reference 23). Using Figure 7 together with formulas (13) and (9), the reflection coefficient and attenuation for the portion of the plasma sheath of interest around a reentering object can be estimated.

Another quantity needed is the thickness of this portion of the plasma sheath. To provide a means for estimating this thickness for the Apollo Command Module, pending accurate

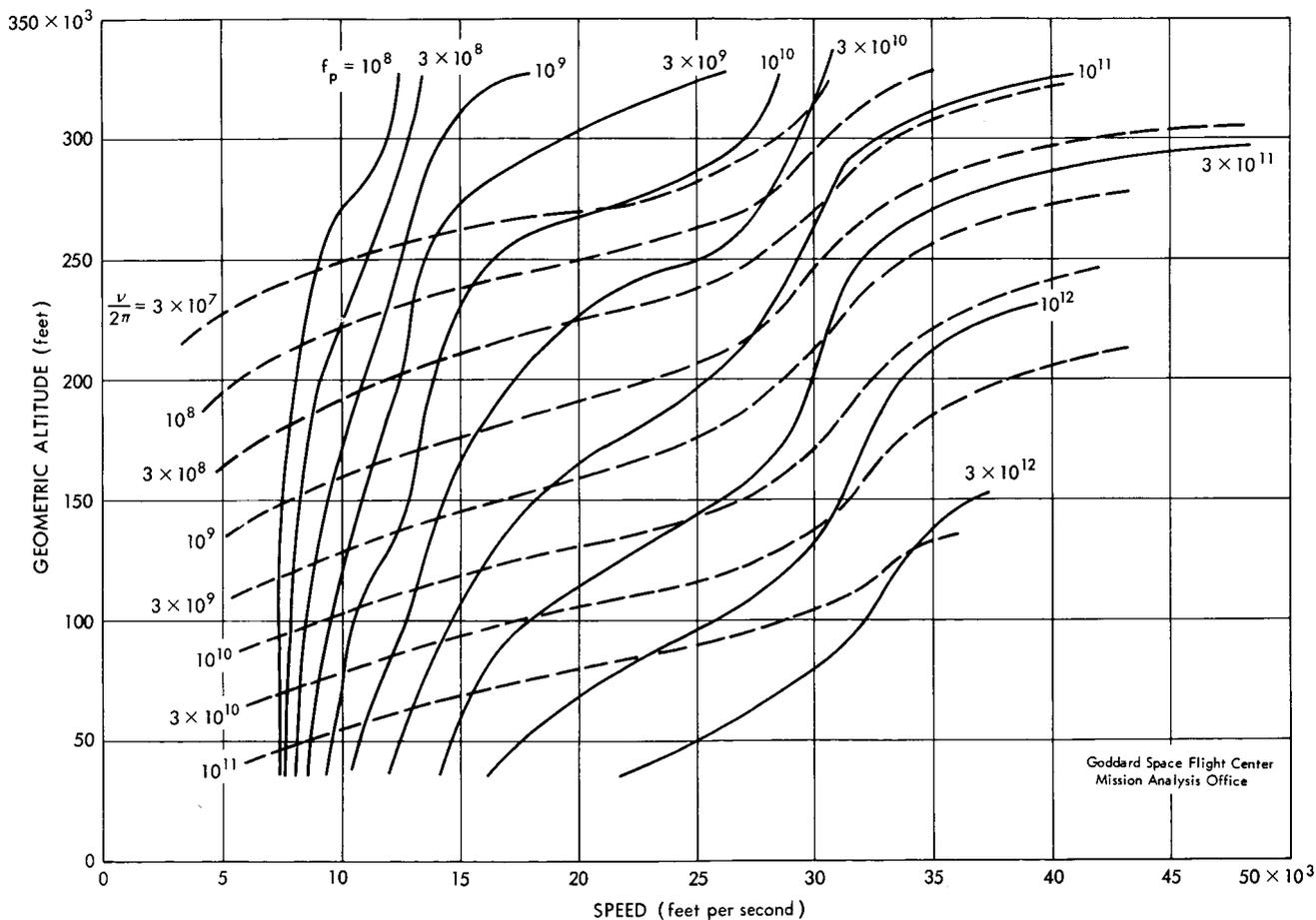


Figure 7—Plasma Frequency and Collision Frequency versus speed and altitude for equilibrium normal-shock flow.

calculations, the shock detachment distance in front of a reentering sphere will be used. The distance Δ has been given by Serbin as (Reference 24)

$$\Delta = 2r/3(K - 1) \quad (1)$$

where r is the radius of the sphere and K is density ratio across a normal shock.

Using the values of K given in Reference 19, the shock detachment distance per unit radius (Δ/r) has been plotted in Figure 8 as a function of altitude and speed. The sheath thickness in front of the Apollo Command Module is estimated taking the indicated percentage of the value $r = 470$ cm, which is the radius of curvature of the blunt face of the Module.

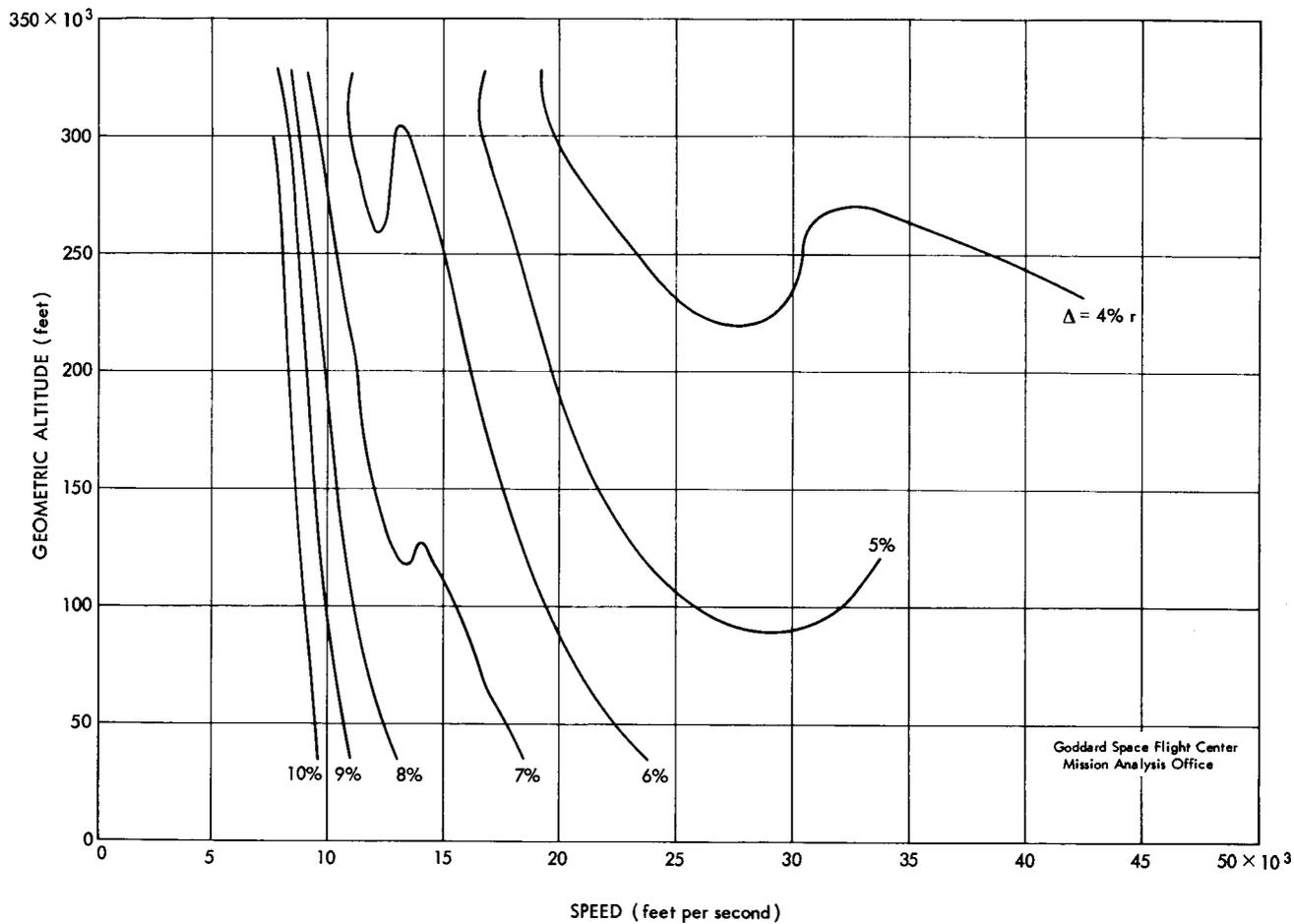


Figure 8—Shock detachment distance Δ versus altitude and speed for a sphere. Δ is given as a percentage of the radius r of the sphere.

BOUNDS ON THE HIGH ALTITUDE INTERFERENCE DIP

Bounds on radar cross section reduction caused by the "capped sphere" (deformation-type) mechanism at high altitudes will be given in this section.

In order to provide a concrete example with which to work, the plot of the Apollo 202 reentry is shown in Figure 9. Initially, the Module descends at an almost constant (actually slightly increasing) speed at altitudes where the atmosphere is tenuous. Assuming a C-band radar frequency (3 GHz), it is seen that the normalized plasma frequency f_p is equal to one at about 310,000 feet. Above this height, f_p is less than one, and below it, greater than one. Throughout most of this part of the trajectory the normalized collision frequency q is small, and can be neglected. For sufficiently small values of f_p , it follows from equation (1) that ϵ is nearly unity and the plasma is not strong enough to effect the radar wave. For large values of f_p , $\epsilon \approx -f_p^2$, $\beta = 0$, and $\alpha = f_p$. It follows from (12) that $R \approx 1$, and the plasma is a good reflector. Consequently, the nominal value $f_p = 1$ will be taken as a high altitude bound for the occurrence of the deformation-type dip.

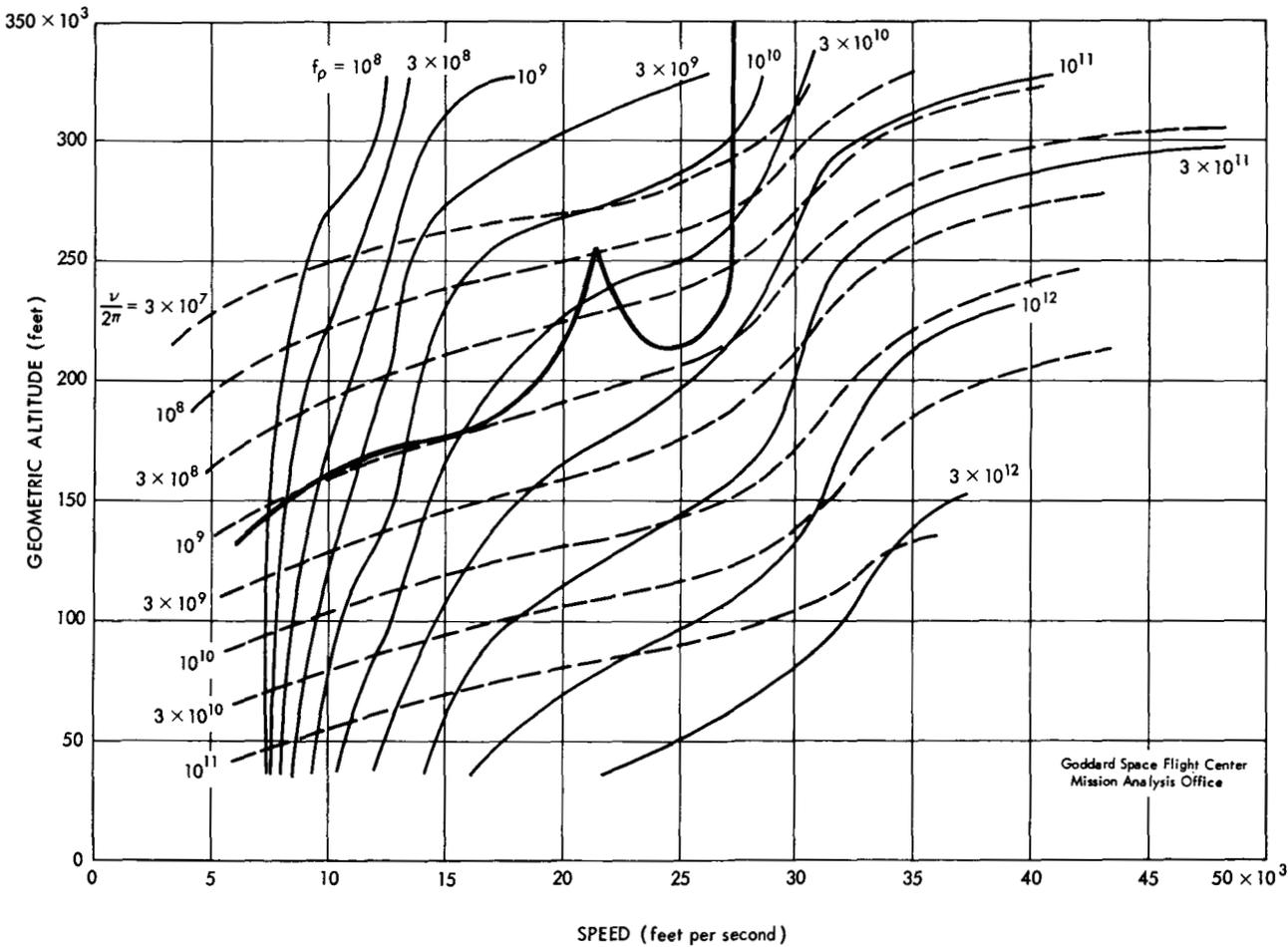


Figure 9—AS-202 Reentry.

As the Module descends, the overdense region will spread out to eventually cover the entire nose, and beyond this point cross section decrease of the type predicted by the model cannot occur. To approximately locate this lower bound, it is only necessary to note that the presence of an overdense plasma sheath extending past the corners of the Module and over the antennas of the Module is the cause of communication blackout. The region where an interference-type reduction can take place, therefore, lies somewhere between the curve $\rho = 1$ and the curve for C-band communications blackout. Both of these curves are plotted in Figure 10, the latter curve being taken from Reference 2.

The bounds shown are probably wider than necessary. However, they do serve to bring out an important, if obvious, point. This is that the deformation-type cross section decrease will not be a problem to a radar that tracks initially using a beacon, and then switches over to skin-tracking after the beacon return becomes blacked out. The skin-return cross-section decrease will have taken place prior to blackout of the beacon signal.

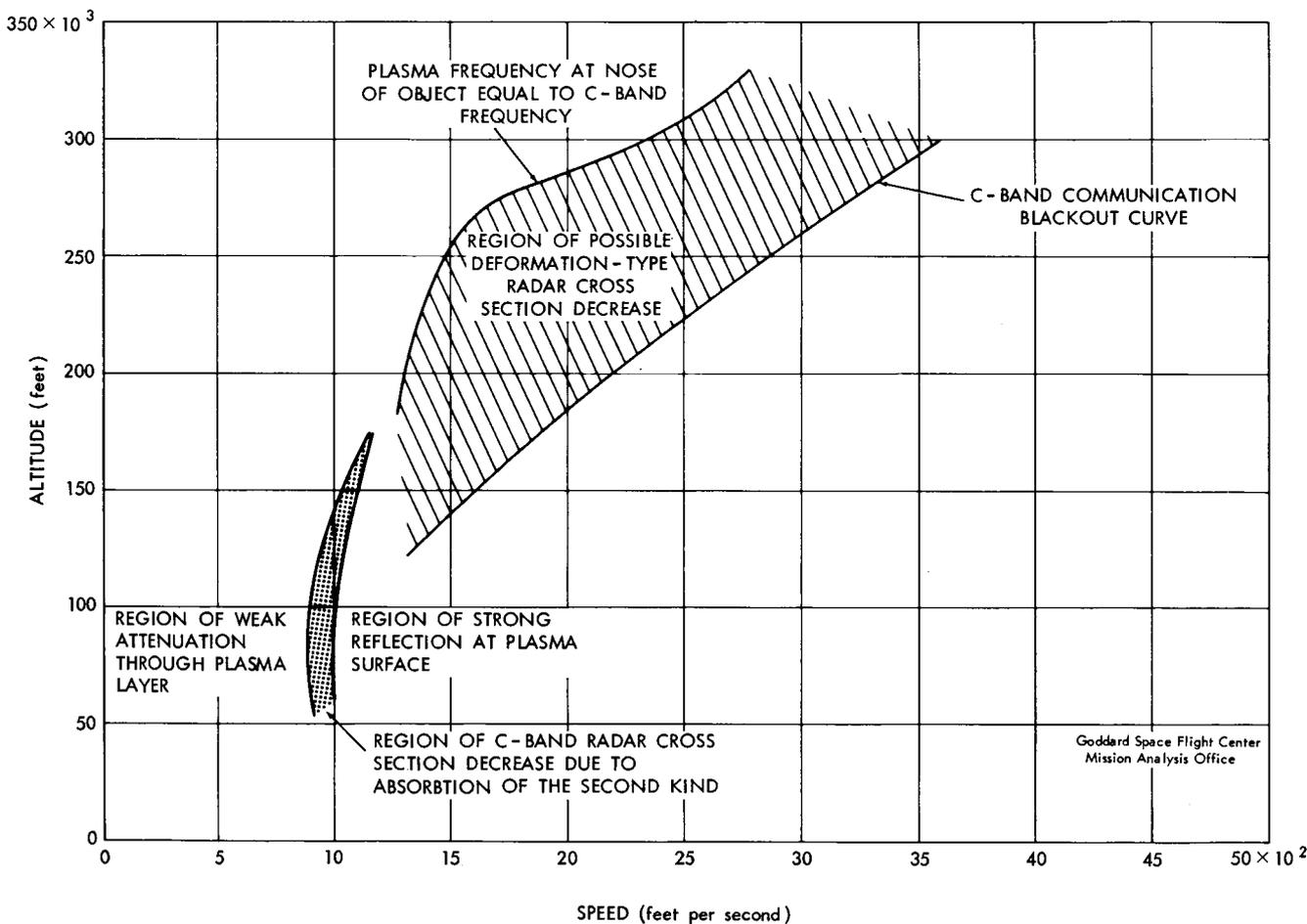


Figure 10—Some bounds on two types of decrease of the cross section seen by C-band radar viewing a large blunt directly-approaching object.

BOUNDS ON THE LOW ALTITUDE REDUCTION DUE TO ABSORPTION OF THE SECOND KIND

As mentioned in section III, a plasma layer can act as an interference absorber or, imperfectly, as an absorbant material of the second kind. In the last case there are two requirements on the condition of the plasma. In the first place the basic reflection coefficient of the air-plasma interface must be low. Second, the plasma attenuation constant must be large enough to attenuate sufficiently the wave transmitted into the plasma and reflected by the body surface. Using these conditions, bounds can be obtained on the region of the altitude-speed diagram where radar absorption of second kind occur.

If equation (11) is solved for $\sqrt{\epsilon}$

$$\sqrt{\epsilon} = (1 - \rho)/(1 + \rho) \quad (14)$$

Since the magnitude of the complex voltage reflection coefficient ρ is assumed small

$$\begin{aligned} \sqrt{\epsilon} &= (1 - \rho)(1 - \rho + \rho^2 - \dots) \\ &\doteq 1 - 2\rho \end{aligned} \quad (15)$$

Squaring

$$\epsilon \doteq 1 - 4\rho \quad (16)$$

From equations (1) and (16)

$$\rho = \frac{1}{4} \cdot \frac{p^2}{1 + q^2} + j \frac{q}{4} \frac{p^2}{1 + q^2} \quad (17)$$

$$|\rho| \doteq \frac{1}{4} \frac{p^2}{\sqrt{1 + q^2}} \quad (18)$$

From equation (5), α equals the negative of the imaginary part of the $\sqrt{\epsilon}$. This, from (15), is twice the imaginary part of ρ , or

$$\alpha \doteq \frac{q}{2} \frac{p^2}{1 + q^2} \quad (19)$$

Let it be stipulated that the reflection coefficient $|\rho|$ must be reduced by more than some factor F in the region of radar absorption decrease. Then (18) gives

$$\frac{4}{F} > \frac{p^2}{\sqrt{1+q^2}} \quad (20)$$

Let it also be stipulated that the wave propagated through the plasma thickness Δ and back (a distance 2Δ) must also be attenuated by at least the same fraction. Then in view of equation (4)

$$\frac{1}{F} > e^{-\alpha 4\pi\Delta/\lambda} \quad (21)$$

or

$$\ln F < 4\pi\alpha\Delta/\lambda \quad (22)$$

From the latter and (19)

$$\frac{\lambda \ln F}{2\pi\Delta} < q \frac{p^2}{1+q^2} \quad (23)$$

Equations (20) and (23) may be combined to give, finally

$$\frac{\ln F}{2\pi} \frac{\lambda}{\Delta} < \frac{q p^2}{1+q^2} < \frac{4}{F} \frac{q}{\sqrt{1+q^2}} \quad (24)$$

If F is arbitrarily set equal to 10.6 (a 21 db reduction) equation (24) becomes

$$\frac{\lambda}{\Delta} < 2.6 \frac{q}{1+q^2} p^2 < \frac{q}{1+q^2} \quad (25)$$

The inequality (25) defines the region of radar cross section reduction due to absorption of the second kind.

The right hand side of the inequality is always less than unity, no matter what value q has. Thus the shock wave thickness Δ must exceed one wavelength (for the value of F assumed). If both λ and Δ are given, the inequality sets a lower bound on the value of q , and provides, for each value of q greater than this lower limit, a possible range of values for p .

Using Figures 7 and 8, the region of the altitude-speed diagram that satisfies inequality (25) is plotted in Figure 10. Radar cross section reduction due to absorption is to be expected in this region. If the theoretical treatment above held exactly, the magnitude of the decrease in this region would always be greater than the in-phase sum (15db down) of two waves each of which is 21db down from the incident wave. Actually, the simple model used does not correspond closely enough to the physical situation to make close quantitative agreement likely.

CONCLUSIONS

The exact calculation of radar cross sections during reentry is a very difficult problem. Nevertheless, it is possible in some cases to make simple estimates of the regions in a reentry altitude-speed diagram where decreases in radar cross section are likely to occur. This has been done in a tentative fashion for the decrease due to two of the four mechanisms likely to cause a decrease, and the results are given in Figure 10 for a C-band radar viewing an object the size of the Apollo Command Module head on. These results are based on grossly simplified models of both the conditions in the plasma sheath and also of the electromagnetic aspects of the problem. Considerably better definition of these regions should be possible through the use of more accurate estimates of the conditions in the sheath and through the use of more sophisticated treatment of the electromagnetic aspects of the problem. Experimental verification and empirical modification of the regions through the use of field data is desirable.

Neither the mechanism of interference absorption nor that of precursor photo-ionization, which might be important because of the high Apollo reentry speeds, has been treated. It is intended to study these effects and report on them at a later date. The determination of the effect of reentry on cross-sections observed at other than small values of the look-angle requires a detailed knowledge of the electromagnetic properties of the plasma sheath.

Because of the incomplete and tentative nature of the analyses and because of the lack of field data on reentries of the Apollo type, it is premature to say whether reentry cross-section decrease is a practical tracking problem.

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